(Generalized) Mahler measures in complex projective space and zeta values

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We study logarithmic and generalized Mahler measures of linear polynomials in complex projective space $\mathbb{CP}^n$ using spectral methods. Starting from the identity relating (spectral) Green's functions and heat kernels on $\mathbb{CP}^n$, we exploit the spectral decomposition of the Laplacian and properties of the Radon transforms along hyperplanes to obtain explicit evaluations of Green's currents associated to divisors of linear forms. This yields convergent series expansions for Mahler measures with computable error bounds, including new (amusing) representations of $\zeta(3)$. Generalized Mahler measure, following Chambert-Loir and Thuillier, is viewed as an integral of Green's current (associated to divisors of linear forms) against normalized Fubini-Study measure on iterated hyperplane sections. We prove that the integral of the generalized Mahler measure along the entire space ($\mathbb{CP}^n$) can be expressed in terms of the rational linear combinations of either even or odd zeta values, with parity of the dimension n and the number of iterations dictating whether odd or even zeta values appear. These results extend the arithmetic interpretation of Mahler measures, and highlight the interplay between spectral theory, projective geometry, and special values of zeta functions.
The talk is based on the joint work with Jim Cogdell, Jay Jorgenson and Anne-Maria Ernvall-Hytonen.