Counting theorem relying on algebraic independence criterion

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This is a joint work with Gal Binyamini, Makoto Kawashima and Yuval Salant.

In this talk, we consider one generalization of the Pila-Wilkie theorem. Let X be a manifold-like set, definable in an O-minimal structure in a certain sense. We study a problem of counting the number of points where X intersects algebraic varieties V over $\overline{\mathbb{Q}}$ of dimension $k < \operatorname{codim} X$, by a function of $T := \deg V + \mathrm{h}(V)$, where $\mathrm{h}(V)$ is the log-height of V. After removing a suitable algebraic part, it is partly proven that this number grows polynomially in T.

We show then, adapting a modification of an algebraic independence criterion proven by P. Philippon in 1986 (improved by C. Jadot in 1996) with particular X, that all intersections above are contained in the collection of a polynomial amount in T, of balls of radius e^{-T}.

We explain how Philippon's algebraic independence criterion can be adapted to such counting.