NAP 2019 - MODULE V - CLASS #8
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We always assume $char(K) \neq 2, 3$.

- We outlined an algorithm to solve by radicals an irreducible polynomial of degree 4 belonging to $K[X]$:
$$f(X) = X^4 + aX^3 + bX^2 + cX + d.$$
  - by the change of variables $X = Y - \frac{a}{4}$ we obtain the depressed form
$$g(Y) = Y^4 + pY^2 + qY + r.$$
  - use the resolution method for cubic polynomials in order to find the roots A, B, C of the cubic resolvent:
$$h(X) = X^3 + 2pX^2 + (p^2 - 4r)X - q^2.$$
  - choose square roots β, γ, δ of A, B, C respectively, respecting the condition $\beta\gamma\delta = -q$;
  - then the roots $\alpha'_1, \alpha'_2, \alpha'_3, \alpha'_4$ of $g(Y)$ can be computed by
$$\alpha'_1 = \frac{1}{2}(\beta + \gamma + \mu)$$
$$\alpha'_2 = \frac{1}{2}(\beta - \gamma - \mu)$$
$$\alpha'_3 = \frac{1}{2}(-\beta + \gamma - \mu))$$
$$\alpha'_4 = \frac{1}{2}(-\beta - \gamma + \mu).$$
  - The roots $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ of $f(X)$ are obtained by putting $\alpha_i = \alpha'_i + \frac{a}{4}$, for $i = 1, 2, 3, 4$.
- We computed the Galois group for irreducible quartic polynomials (exercise 14.7 of Garling's book).
- To conclude, we gave the definition of *solvable group*, and stated the results in Chapter 17 of Garling's book, relating the solvability by radicals of a polynomial to the solvability of its Galois group.