NAP 2019 - MODULE V - CLASS #4
July 19, 2019

Lea Terracini

- **Lemma**: Let G be an abelian group and $g_1, g_2 \in G$ be such that $o(g_1) = n, o(g_2) = m$ are coprime integers. Then $o(g_1 g_2) = nm$.

- **Example**: Let $K = \mathbb{F}_5(\alpha)$, where α is a root of $X^2 - 2$. We found that $o(\alpha) = 8$, $o(\alpha + 1) = 12$. Then $(\alpha + 1)^4$ has order 3, and by the previous lemma $\alpha(\alpha + 1)^4 = 2\alpha + 4$ is a generator of $K^\times$.

- Let $(G, +)$ be an abelian group. The set T of elements of finite order in G is a subgroup, called the *torsion* of G.

- Let $(G, +)$ be finitely generated abelian group. It is said *free* (of rank s) if it is isomorphic to $\mathbb{Z}^s$ for some $s \in \mathbb{N}$.
  Fact: Let G be a finitely generated abelian group and let $g_1, \ldots, g_s$ be a minimal set of generators. Then

  $$G \text{ is free if and only if } \sum_i a_i g_i = 0 \Rightarrow a_1 = a_2 = \ldots = a_s = 0,$$

  i.e. if and only if $g_1, \ldots, g_s$ are linearly independent over $\mathbb{Z}$.

- **Classification theorem of finitely generated abelian groups:** every f.g.a.g. is isomorphic to the product of a free group of finite rank and a finite group:

  $$G \simeq \mathbb{Z}^s \times T, \qquad \text{where } T \text{ is a finite group.}$$

- **Introduction to the subject of next classes:** we gave an intuitive notion of a *solution by radicals* of a polynomial equation. In order to make it precise, we introduced the definitions of

  - *radical* element in a field extension L/K;
  - *extension by radicals*
  - polynomial *solvable by radicals*

  as in §14.1 of Garling book.