NAP 2019 - MODULE V - PROBLEM SET 2
DUE TUESDAY JULY 30, 2019, AT 24:00 KATHMANDU TIME

Exercise 1

Show that if the Galois group of a rational cubic polynomial $f(x)$ is cyclic of order 3 then $f(x)$ has only real roots.

Exercise 2

Let f be an irreducible cubic polynomial over a finite field of characteristic different from $2, 3$. Show that its discriminant is a square.

Exercise 3

Let K be a subfield of $\mathbb{C}$. Let $f(x) = x^3 + px + q$ be an irreducible polynomial in $K[x]$. Let α be a root of $f(x)$. Let $\beta = a + b\alpha + c\alpha^2 \in K(\alpha) \setminus K$, with $a, b, c \in K$. Determine the minimal polynomial $g(X)$ of β over K. Let Δ be the discriminant of $f(X)$ over K. Show that $K(\alpha)/K$ is an extension by radicals if and only if -3Δ is a square in K.

Exercise 4

Solve Exercise 14.5 in Garling's book.

Exercise 5

For each of the following polynomials, check irreducibility and give the Galois group over $\mathbb{Q}$:

a) $X^3 - 4X - 1$;
b) $X^4 + X + 1$;
c) $X^4 + X^3 + X^2 + X + 1$;
d) $X^4 + X^3 - X^2 - 2X - 2$.

Please justify all answers!