The proof of the Main Theorem of Galois theory.
Let $\mathbf{K} \subset \mathbf{E} \subset \mathbf{L}$ be field extensions, denote by G the G;laois group of $\mathbf{K} \subset \mathbf{L}$. The statements
(1) $\#Aut_{\mathbf{E}}(\mathbf{L}) = [\mathbf{L} : \mathbf{E}]$
(2) $\#H = [\mathbf{L} : \mathbf{L}^G]$
imply that
$\#Aut_{\mathbf{K}}(\mathbf{L}) = [\mathbf{L} : \mathbf{K}] = [\mathbf{L} : \mathbf{L}^G]$, hence $\mathbf{K} = \mathbf{L}^G$.
Statements (1) and (2) imply the Galois correspondence

$$\{\mathbf{K} \subset \mathbf{E} \subset \mathbf{L}\} \quad \leftrightarrow \quad \{H \subset G\}$$
$$\mathbf{E} \to Aut_{\mathbf{E}}(\mathbf{L})$$
$$L^H \leftarrow H.$$