The Fundamental Theorem of Galois Theory.
(cf. Garling, Thm. 11.8, page 97).

- Recalled the basic notions involved in the statement of the Theorem.
- Discussed the statement of the theorem and its significance.
- Examples of the Galois correspondence:

(1) $\mathbf{R} \subset \mathbf{C}$
$Aut_{\mathbf{R}}(\mathbf{C}) = \{id, \phi(a+ib) = a - ib\} \cong \mathbf{Z}_2$.

(2) $\mathbf{Q} \subset \mathbf{Q}(\sqrt[3]{2}, \omega)$, where $\omega = e^{2\pi i/3} = \frac{1}{2}(-1+\sqrt{-3})$ is a primitive cube root of 1.
$[\mathbf{Q}(\sqrt[3]{2}, \omega) : \mathbf{Q}] = 6$;
$G := Aut_{\mathbf{Q}}(\mathbf{Q}(\sqrt[3]{2}, \omega)) \cong \mathcal{S}_3$.

The subgroups H of G are:

$$1, \quad \mathcal{S}_3, \quad A_3 = \langle (123), (132) \rangle, \quad \langle (12) \rangle, \quad \langle (13) \rangle, \quad \langle (23) \rangle.$$

The subgroup A_3 has index 2 in $\mathcal{S}_3$ and its fixed field $\mathbf{Q}(\omega)$ has degree 2 over $\mathbf{Q}$;
Each of the subgroups generated by a 2-cycle has index 3 in $\mathcal{S}_3$ and its fixed field has degree 3 over $\mathbf{Q}$. These subfields are $\mathbf{Q}(\sqrt[3]{2})$, $\mathbf{Q}(\sqrt[3]{2}\omega)$, $\mathbf{Q}(\sqrt[3]{2}\omega^2)$.