Due Tuesday July 16, 2019, at 24:00 Kathmandu time

1. Consider the field $K = \mathbf{Q}(\zeta_5, \sqrt[3]{2})$.
   (a) Show that K admits subfields of degrees 3 and 4.
   (b) Determine $[K : \mathbf{Q}]$.

2. Consider the field $K = \mathbf{Q}(\zeta_{12}, \sqrt[4]{3})$.
   (a) Show that K admits at least two distinct subfields of degree 4.
   (b) Determine $[K : \mathbf{Q}]$.

3. Consider the field $K = \mathbf{Q}(\zeta_{13})$.
   How many distinct subfields of K are there?

Justify all answers!