Due Tuesday July 9, 2019, at 24:00 Kathmandu time

1. Consider the extension $\mathbf{Q} \subset \mathbf{Q}(\sqrt{2}, \sqrt{3})$.

(a) Prove that its Galois group over $\mathbf{Q}$ is isomorphic to the group $\mathbf{Z}_2 \times \mathbf{Z}_2$.

(b) Enumerate the subgroups H of $\mathbf{Z}_2 \times \mathbf{Z}_2$.

(c) Describe the Galois correspondence between subgroups of $\mathbf{Z}_2 \times \mathbf{Z}_2$ and subfields of $\mathbf{Q}(\sqrt{2}, \sqrt{3})$.

2. Let ζ_9 be a primitive ninth root of unity.

(a) Show that ζ_9 is a zero of $f(X) = (X^9 - 1)/(X^3 - 1) = X^6 + X^3 + 1$ and show that $f(x)$ is the minimum polynomial of ζ_9 over $\mathbf{Q}$.

(b) Show that $\mathbf{Q} \subset \mathbf{Q}(\zeta_9)$ is a Galois extension with Galois group $\mathbf{Z}_9^*$.

(c) Enumerate the subgroups H of $\mathbf{Z}_9^*$.

(d) Describe the Galois correspondence between subgroups of $\mathbf{Z}_9^*$ and subfields of $\mathbf{Q}(\zeta_9)$.