

NAP 2019, MODULE-III, EXERCISE SET 2: DEADLINE 18 JUNE, 2019

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- (1) Exercise 9.2 from the book.
- (2) Exercise 9.3 from the book.
- (3) Prove Exercise 9.4 of the book in the case $L : K$ is finite.
- (4) Let $K \subseteq M \subseteq L$ be an extensions of fields such that $M : K$ and $L : M$ is normal. Is $L : K$ normal ? If so, prove this or else give a counter-example.
- (5) Find the normal closure for the following field extensions: (a) $\mathbb{Q}(\sqrt[p]{2}) : \mathbb{Q}$ where p is a prime; (b) $\mathbb{Z}_3(\alpha) : \mathbb{Z}_3$ where $\alpha^3 - \alpha + 1 = 0$
- (6) Let $L : K$ be a finite normal extensions. Prove that the number of automorphisms of L which fixes K is at most $[L : K]$.
- (7) Let $L : K$ be algebraic. Suppose that $\alpha, \beta \in L$ are separable over K . Prove that $\alpha + \beta$ and $\alpha\beta$ are separable over K .
- (8) Exercise 10.1 from the book.
- (9) Find the number of automorphisms of $\mathbb{Q}(\sqrt[3]{2}, \omega)$ which fixes \mathbb{Q} .
- (10) Consider an extension $\mathbb{Z}_p(t^{1/p}) : \mathbb{Z}_p(t)$ where p is a prime and t an indeterminate over \mathbb{Z}_p . Prove that the number of automorphisms of $\mathbb{Z}_p(t^{1/p})$ which fixes $\mathbb{Z}_p(t)$ is less than p .