NAP 2019, MODULE II, CLASS #5, MAY 29, 2019

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- Recall the definition of a unit and of an irreducible element in a ring. Example: K[x].
- ullet Irreducibility of polynomials over a field K; degree :
 - 0, reducible (unit)
 - 1, irreducible
 - 2, irreducible if and only if no root in K; solving the quadratic equation, discriminant Δ . Criterion for irreducibility: Δ not a square in K;
 - 3, irreducible if and only if no root;
 - \geq 4, examples of $(x^2+1)^2$, $(x^2+1)(x^2+2)$ reducible in $\mathbb{R}[x]$ but no root in \mathbb{R} .
- Recall : irreducibility in $\mathbb{Z}[x]$.
- Existence of an algorithm for the irreducibility in $\mathbb{Z}[x]$.
- Reduction modulo p of a polynomial f in $\mathbb{Z}[x]$. If one reduction modulo a prime p is irreducible and of the same degree as f, then f is irreducible over \mathbb{Z} .