NAP 2019, MODULE II, CLASS #3, MAY 24, 2019

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• Extension of K generated by A; notation: $K(A)$. Simple extension $K(\alpha) : K$. Finitely generated extension $K(\alpha_1, \dots, \alpha_n) : K$.
Exemples.

• Algebraic and transcendental elements (continued): for an extension $L : K$ and an element $\alpha \in L$, evaluation map

$$\begin{array}{rcc} E_\alpha : K[X] & \to & L \\ P & \mapsto & P(\alpha). \end{array}$$

• Element $\alpha \in L$ transcendental over K, $\ker E_\alpha = \{0\}$. Isomorphisms $K[\alpha] \simeq K[X]$, $K(\alpha) \simeq K(X)$ as rings (fields) and K–vector spaces.

• Element $\alpha \in L$ algebraic over K; $\ker E_\alpha = (m_\alpha)$: minimal polynomial $m_\alpha \in K[X]$ of α. Irreducibility of m_α in $K[X]$. Degree of an algebraic element.

Theorem 4.3: $K[\alpha] = K(\alpha) = K[X]/(m_\alpha)$.

Theorem 4.4 : $\alpha \in L$ is algebraic over K if and only if $[K(\alpha) : K] < \infty$. Equality of the degrees.

Theorem 4.5 : Given an extension $L : K$, the set of elements in L algebraic over K is a subfield of L.