NAP 2019, MODULE II, HOMEWORK ASSIGNMENT #1
DUE TUESDAY, MAY 28

Michel Waldschmidt

• **Homework assignment** (due Tuesday, May 28, 10pm Kathmandu Time).

1. Solve problems
4.1 p. 42,
4.3, 4.4, 4.6 p. 45.
4.7, 4.8 p. 47,
4.9, 4.10, 4.11 p. 48.

2. Solve problem 4.2 p. 45.
`Hint:`
(a) First prove that if $L : K$ is an algebraic extension and A a subset of L which is a ring and a K–vector space, then A is a subfield of L.
(b) Consider the subset A of L of finite sums $x_1y_1 + \cdots + x_my_m$ where $x_i \in K_1$ and $y_i \in K_2$.
Prove that A is a subring of L.
(c) Prove that A is a subspace of the K–vector space L, of dimension $\leq [K_1 : K][K_2 : K]$.
(d) Conclude.

3. Let j be a complex root of $X^2 + X + 1$ (notice that $j^3 = 1$ and $j \neq 1$), let $\alpha = j + \sqrt[3]{2}$, $K = \mathbb{Q}$, $L = \mathbb{Q}(j)$. Check that the numbers $[L(\alpha) : L]$ and $[L : K]$ are relatively prime and that the coefficients of the minimal polynomial of α over L are not in K.
Compare with problem 4.5 p. 45.