NAP 2019, CLASS #1, MAY 06, 2019

ROGER & SYLVIA WIEGAND

This year we have a new book, "Galois Theory", by D. J. H. Garling. To download the book, go to the NAP website: http://www.rnta.eu/nap/nap-2019/ and click on "2019 Edition". Now select "Didactics" and click on "here". (By the way, it will be worthwhile for you to browse this website from time to time.)

Warning: Some of the notation in the book is a bit different from standard usage. We will try to point out such aberations, particularly when we use notation different from that in the book. One particular point is that the book invariably uses "$f \circ g$" to indicate the composition of two functions; this means that g acts first, that is, $(f \circ g)(x) = f(g(x))$. In particular, when working with permutations, Garling seems to think that $(12)(13) = (132)$, so (13) acts first. Most people, however, would assume that (12) acts first, so the product would be (123). We'll go along with Garling's notation, but be aware that in future modules the instructors may revert to the more standard convention with permutations.

Module 1, taught by Roger Wiegand and Sylvia Wiegand, is proposed to cover Chapters 1, 3, and 5 of Garling's book. Technically much of this material is intended to be review for you, and we do not have time to go into all of it in detail. We will cover some of the highlights, but you should read all of the material carefully on your own and bring questions to us, if you have them.

Some items covered in Class #1, May 06, 2019:

• Definition of "group". Examples: symmetries of the triangle, permutations, $\mathbb{Z}, +$

• Right cosets.

• Lagrange's Theorem: If G is a group and $H \leq G$, then $|G| = |H| \cdot |G/H|$; proof

• Homomorphisms of groups; isomorphism

• kernel of a homomorphism ("$\operatorname{Ker} g$" denotes the kernel of g). A homomorphism g is one-to-one if and only if $\operatorname{Ker} g = \{1\}$.

• Normal subgroups (motivation in terms of forming quotient group), various equivalent formulations:

(1) $gHg^{-1} \subseteq H, \forall g \in G$.
(2) $g^{-1}Hg \subset H, \forall g \in G$.
(3) $gHg^{-1} = H, \forall g \in G$.
(4) $Hg = gH, \forall g \in G$.

• Notations:

(1) N is a subgroup of G: $N \leq G$.
(2) N is a normal subgroup of G: $N \lhd G$

• If $\varphi : G \to H$ is a homomorphism, then $\operatorname{Ker} \varphi \lhd G$.

• Quotient group G/N when $N \lhd G$

• The First Isomorphism Theorem (slightly jazzed up): Let $N \lhd G$, and let $\varphi : G \to H$ be a group homomoprhism. Let $\pi : G \twoheadrightarrow G/N$ be the map taking g to the coset Ng. Then there is a homomorphism $\bar{\varphi} : G/N \to H$ such that $\overline{\varphi}\pi = \varphi$ if and only if $N \subseteq \operatorname{Ker} g$. In this case the homomorphism is unique and satisfies

$\overline{\varphi}(Ng) = \varphi(g)$ for every $g \in G$. Moreover, $\operatorname{im}\overline{\varphi} = \operatorname{im}\varphi$, and $\operatorname{Ker}\overline{\varphi} = \frac{\operatorname{Ker}\varphi}{N}$. In particular, taking $N = \operatorname{Ker} g$, we see that $\overline{\varphi}$ is an isomorphism from $G/\operatorname{Ker}\varphi$ onto $\operatorname{im}\varphi$.

Reading assignment, Week 1: pp. 3-13, & 18-27. Week 2: pp. 27-36 & 47-53.

Class procedures

1. You are encouraged to discuss problems with each other and to ask the instructors and tutors questions about them, but hand in your own version of the solution. Your write-up should be done entirely on your own.

2. Please don't browse the internet for solutions to problems.

3. Please ask questions of the teacher in class.

4. Please don't talk in class, and please put your cell phones away. But, once again, *do* ask questions. If something seems unclear please let us know.

5. Read the material to be covered in class *both before and after* that class.

Problem Set #1 . This assignment must be completed and submitted to Nilu (Dr. Nilakantha Paudel) by 10 pm Kathmandu time on Monday, 13 May, 2019. To be sure it gets there, please use both of these email addresses: paudel@mat.uniroma3.it and nilu.paudel@gmail.com. If you misplace his email addresses you can recover them by going to the NAP website and select 2019 Edition / Didactics / Dr. Nilakantha Paudel [Nilu]. Meanwhile, you should figure out the best way to prepare your solutions so that they can be sent by email. One way is to write them up by hand, neatly, and scan them to a file. A better solution is to use TeX or LaTeX to prepare them. (It's probably worthwhile to learn to use TeX. One can't get rich and famous doing math without knowing how to use TeX.)

Problems from the book (pages 7, 9, and 13): numbers 1,3,4,6,12,13, 16, 17 (modified as indicated below). Be sure to prove all of your assertions in detail.

Problem 17, modified: (a) The problem in the book is a bit tricky, so just do it for a two-dimensional vector space V over an infinite field K. (b) Show that if K is the two-element field, then every two-dimensional vector space over K is the union of three proper subspaces.