

CORRECTIONS AND COMMENTARY FOR PROBLEM SET #2

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In several of the problems you will need the following fundamental theorem, which appears not to be stated explicitly in the book:

Fundamental Theorem on Degrees of Simple Extensions (FTDSE): Let K/F be a field extension, and let $\alpha \in K$ be an element that's algebraic over F , with minimal polynomial $f(X) \in F[X]$. Then $[F(\alpha) : F] = \deg f(X)$.

Just quote this Fundamental Theorem whenever you need it.

Problem 1: The hint in part (c) does not seem to be useful, so it might be best to ignore it.

Problem 2: In part (b), don't write more than a few lines. Most of the details are in the footnote.

Problem 3: The last sentence should say "Prove that $F(\alpha^2) = F(\alpha)$."

Problem 6: "Prove that $d = [F(\alpha)]$ if and only if d is the" should be "Prove that $d = [F(\alpha) : F]$ if and only if $f(X)$ is the".

Problem 7: It might help to do (b) before you do (a).

Problem 8: The " \Leftarrow " implication holds in any commutative ring. The " \Rightarrow " implication can fail without the "domain" hypothesis.