

- Answer to questions dealing with Problem Set 1.

- Constructions with straight edge and compass (Milne p. 20-23)

Definitions. Examples: the set \mathcal{C} of constructible numbers contains \mathbb{Q} . Construction of \sqrt{c} for $c \in \mathcal{C}$.

Theorem 1.36: the set of constructible numbers is a field; a number α is constructible if and only if it belongs to a subfield of \mathbb{R} of the form $\mathbb{Q}(\sqrt{a_1}, \sqrt{a_2}, \dots, \sqrt{a_r})$ where $a_1 \in \mathbb{Q}$ and for $i = 2, \dots, r$, $a_i \in \mathbb{Q}(\sqrt{a_1}, \sqrt{a_2}, \dots, \sqrt{a_{i-1}})$.

The field obtained by adjoining to \mathbb{Q} the square roots of prime numbers is the same as the field obtained by adjoining all the square roots of positive rational numbers (*It took some time to convince the students that the two fields are the same*).

Example of a constructible number: $\sqrt{(\sqrt{2} + 2)\sqrt{3} + 3}$ (cf. Exercise 3.4).

Corollary 1.37: constructible numbers are algebraic numbers with degree a power of 2.

Corollary 1.38: duplication of the cube.

Corollary 1.39: trisection of the angle; $\cos(\pi/9)$ is not constructible.

Construction of regular polygons. Fermat numbers, Fermat primes.

Reference: J.S. Milne, Fields and Galois Theory Version 4.52 March 17, 2017.