NAP 2018 Module III Lecture 3 Wednesday, June 6, 2018, 18:00 – 19:30.

• Answer to questions:
— *What is the difference between an extension and a splitting field?*
— *In an extension, is it true that the two fields have the same characteristic?*

• Erratum to Remark 3.11 (b) p. 38 in Milne's notes (pointed out by Roger Wiegand). Corrected version, following Hironori Shiga, NAP 2017 Module III Problem 1
`http://www.rnta.eu/nap/nap-2017/course-2017/module_3_hw_1.pdf`

> Let $E/F$ be a separable finite extension and let $G$ be a finite subgroup of $\mathrm{Aut}(E/F)$. Then Proposition 2.7 (a) says $\#\mathrm{Aut}(E/E^G) = [E : E^G]$. On the other hand, by (3.5), we have $\mathrm{Gal}(E/E^G) = G = \mathrm{Aut}(E/E^G)$, therefore $\#G = [E : E^G]$. In particular, when $E/F$ is Galois with Galois group $G$, we have
>
> $$G = \mathrm{Gal}(E/F) = \mathrm{Aut}(E/F) \quad \text{and} \quad \#\mathrm{Gal}(E/F) = [E : F].$$

• For an extension $E/F$ where $E = F(\alpha_1, \ldots, \alpha_m)$, an element of $\mathrm{Aut}(E/F)$ is determined by its values at $\alpha_1, \ldots, \alpha_m$.
In particular, for a simple extension $E = F(\alpha)$, an element of $\mathrm{Aut}(E/F)$ is determined by its values at $\alpha$.
Conjugates of an algebraic element over $F$ (roots of the irreducible polynomial). If $\alpha$ has $r$ conjugates over $F$ in $F(\alpha)$, then $\mathrm{Aut}(F(\alpha)/F)$ has at most $r$ elements.
For a Galois extension $E/F$ of degree $d$, $\mathrm{Aut}(E/F)$ is a group of order $d$.
Examples.

• Statement of the Fundamental Theorem of Galois Theory (Theorem 3.16).

`Reference`: J.S. Milne, Fields and Galois Theory Version 4.52 March 17, 2017.