- $\mathbf{F} \subset \mathbf{E}$ finite field extension; $\mathrm{Aut}_{\mathbf{F}}(\mathbf{E})$ the group of $\mathbf{F}$-field homomorphisms.
- Let G be a subgroup of $\mathrm{Aut}_{\mathbf{F}}(\mathbf{E})$. Denote by $\mathbf{E}^G = \{x \in \mathbf{E} \mid \phi(x) = x,\ \forall \phi \in G\}$ the subset of invariant elements. Then $\mathbf{E}^G$ is a subfield of E (Exercise).

The next two theorems estimate the cardinality of $\mathrm{Aut}_{\mathbf{F}}(\mathbf{E})$ both from above and from below.

Theorem 1. (Milne, Thm.3.2)
(a) $\mathbf{F} \subset \mathbf{E}$ *finite field extension. Then* $\#Aut_{\mathbf{F}}(\mathbf{E}) \leqslant [\mathbf{E} : \mathbf{F}]$*;*
(b) If $\mathbf{E} = \mathbf{F}_f$ *is the splitting field of a separable polynomial* $f \in \mathbf{F}[x]$*, then* $\#Aut_{\mathbf{F}}(\mathbf{E}) = [\mathbf{E} : \mathbf{F}]$

Theorem 2. (Artin's Lemma) (Milne, Thm.3.4) *Let* $\mathbf{E}$ *be a field, and let* G *be a finite subgroup of* $Aut(\mathbf{E})$*. Then* $\#G \geqslant [\mathbf{E} : \mathbf{E}^G]$.