• Defined the *characteristic* of a field $\mathbf{F}$. The characteristic of a field is either equal to 0 or to a prime p.

• Examples of fields of characteristic 0: $\mathbf{Q}$, $\mathbf{R}$, $\mathbf{C}$, $\mathbf{Q}(\alpha)$, ... Every field of characteristic 0 contains a copy of $\mathbf{Q}$.

• Examples of of fields of characteristic p: the field of p elements $\mathbf{F}_p = \mathbf{Z}/p\mathbf{Z}$, the field of p^n elements $\mathbf{F}_{p^n} \cong \mathbf{F}_p[x]/(f)$, where f is an irreducible polynomial of degree n in $\mathbf{F}_p[x]$. Every field of characteristic p contains a copy of $\mathbf{Z}/p\mathbf{Z}$.

Definition. A field $\mathbf{F}$ is perfect if either $char(\mathbf{F}) = 0$ or $char(\mathbf{F}) = p$ and $\mathbf{F}^p = \mathbf{F}$.

• Examples of perfect fields: $\mathbf{Q}$, $\mathbf{R}$, $\mathbf{C}$, $\mathbf{Q}(\alpha)$, $\mathbf{F}_p$, $\mathbf{F}_{p^n}$.

• Examples of *non*-perfect fields: $\mathbf{F} = \mathbf{F}_p(T)$.

Recall that a polynomial is *separable* if it has distinct zeros in any splitting field.

Proposition (Milne, Prop 2.12) *Let* $\mathbf{F}$ *be a field and let* $f \in \mathbf{F}[x]$ *be an irreducible polynomial. Then*
(a) If $char(\mathbf{F}) = 0$*, then* f *is separable;*
(b) If $char(\mathbf{F}) = p$*, then there exists an irreducible separable polynomial* $g \in \mathbf{F}[x]$ *such that* $f(x) = g(x^{p^e})$*, for some* $e \in \mathbf{Z}_{\geqslant 1}$*;*
(c) If $\mathbf{F}$ *is perfect, then* f *is separable.*

Let $\mathbf{F} \subset \mathbf{E}$ be a field extension.

$$\mathrm{Aut}_{\mathbf{F}}(\mathbf{E}) = \{\phi\colon \mathbf{E} \to \mathbf{E},\ \mathbf{F}\text{-field isomorphisms}\}.$$

If the extension has finite degree $[\mathbf{E} : \mathbf{F}] < \infty$, then $\mathrm{Aut}_{\mathbf{F}}(\mathbf{E}) = \mathrm{Hom}_{\mathbf{F}}(E, E)$.