Concluded the proof of Proposition 2.7, in Milne:

Proposition 2.7(b) *Let* $\mathbf{F}$ *be a field, let* $f \in \mathbf{F}[x]$ *be a polynomial. If* $\mathbf{K}$ *and* $\mathbf{K}'$ *are splitting fields of* f *over* $\mathbf{F}$*, then there exists a field* $\mathbf{F}$*-isomorphism* $\phi\colon \mathbf{K} \to \mathbf{K}'$.

Definition. Let $\mathbf{F}$ be a field. A polynomial $f \in \mathbf{F}[x]$ is called separable if in a splitting field of f the zeroes of f are distinct.

Examples:
(a) $f(x) = x(x-1) \in \mathbf{Q}[x]$ is separable;
(b) $f(x) = (x-1)^2 \in \mathbf{Q}[x]$ is not separable;
(c) $f(x) = x^2 + 1 \in \mathbf{R}[x]$ is separable: the splitting field of f is $\mathbf{R}_f = \mathbf{C}$ and there f has distinct zeros $\pm i$.
(d) $f(x) = x^2 + 1 = (x+1)^2 \in \mathbf{Z}/2\mathbf{Z}[x]$ is not separable.

Definition. Let $\mathbf{F}$ be a field. The derivative of a polynomial $f(x) = \sum_{n=0}^{d} a_n x^n$ in $\mathbf{F}[x]$ is defined as follows

$$f'(x) = \sum_{n=1}^{d} n a_n x^{n-1}.$$

(here $n = 1 + \ldots + 1$, where $1 \in \mathbf{F}$ and $nx = x + \ldots + x$).

Excercise. The derivative defined above is linear and satisfies the Leibnitz rule.

Proposition. (Prop.2.13, Milne) *Let* $\mathbf{F}$ *be a field. A polynomial* $f \in \mathbf{F}[x]$ *is separable if and only if* $\gcd(f, f') = 1$.