$\mathbf{F}$ field, $\mathbf{F} \subset \mathbf{E}$ finite extension, $\mathbf{F} \subset \mathbf{L}$ arbitrary extension.
$\mathrm{Hom}_{\mathbf{F}}(E, L) = \{\phi\colon \mathbf{E} \to \mathbf{L},$ field homomorphisms such that $\phi_{|\mathbf{F}} := id_{|\mathbf{F}}\}$

Goal: estimate the cardinality of $\mathrm{Hom}_{\mathbf{F}}(E, L)$.
Started from special case $\mathbf{E} = \mathbf{F}(\alpha)$, where $\alpha \in \mathbf{E}$, with minimal polynomial $f \in \mathbf{F}[x]$. In this case, an $\mathbf{F}$-homomorphism ϕ is completely determined by $\phi(\alpha)$.

Lemma. (Milne, Prop.2.1) $\mathrm{Hom}_{\mathbf{F}}(\mathbf{F}(\alpha), \mathbf{L})$ *is in 1-1 correspondence with the set* $\mathbf{Z}(f) \cap \mathbf{L} = \{z \in \mathbf{L} \mid f(z) = 0\}$. *The correspondence is given by* $\phi \mapsto \phi(\alpha)$.

Corollary. $\#\mathrm{Hom}_{\mathbf{F}}(\mathbf{F}(\alpha), \mathbf{L}) \leqslant \deg(f)$.

In general (Milne, Prop.2.7)

Proposition. $\#\mathrm{Hom}_{\mathbf{F}}(\mathbf{E}, \mathbf{L}) \leqslant [\mathbf{E} : \mathbf{F}]$.

Example. $\mathbf{F} = \mathbf{Q}$, $\alpha = \sqrt{2}$, $\mathbf{L} = \mathbf{R}$.
Then $\#\mathrm{Hom}_{\mathbf{Q}}(\mathbf{Q}(\sqrt{2}), \mathbf{R}) = 2$.
There are 2 possibilities: $\phi(\sqrt{2}) = \pm\sqrt{2}$.

Example. $\mathbf{F} = \mathbf{Q}$, $\alpha = \sqrt[3]{2}$, $\mathbf{L} = \mathbf{R}$.
Then $\#\mathrm{Hom}_{\mathbf{Q}}(\mathbf{Q}(\sqrt[3]{2}), \mathbf{R}) = 1$.
There is only 1 possibility: $\phi(\sqrt[3]{2}) = \sqrt[3]{2}$.

Example. $\mathbf{F} = \mathbf{Q}$, $\alpha = \sqrt[3]{2}$, $\mathbf{L} = \mathbf{C}$.
Then $\#\mathrm{Hom}_{\mathbf{Q}}(\mathbf{Q}(\sqrt[3]{2}), \mathbf{C}) = 3$.
There are 3 possibilities: $\phi(\sqrt[3]{2}) = \sqrt[3]{2},\ \sqrt[3]{2}\omega,\ \sqrt[3]{2}\omega^2$, where $\omega = e^{2\pi i/3}$.