NAP 2018, CLASS #1, MAY 08, 2018

ROGER & SYLVIA WIEGAND

• Brief review of basic notions (groups, rings, and fields)
• Some examples
• Homomorphisms, examples
• Equivalence relations and equivalence classes.
• Examples: congruence ("clock arithmetic")
• Integral domains, ideals
• Division Algorithm (with proof) for $R[X]$ when R is a commutative ring and the divisor is monic. Note over a field any non-zero polynomial can be made monic by multiplying by a constant.
• Division algorithm for $\mathbb{Z}$.
• Consequences of the division algorithm: $F[X]$ is a principal ideal domain (PID) if F is a field. $\mathbb{Z}$ is a PID.
• Example using the division algorithm for polynomials.

• We passed out the following stuff:

Class procedures

1. You are encouraged to discuss problems with each other and to ask the instructors and tutors questions about them, but hand in your own version of the solution. Your write-up should be done entirely on your own.

2. You are discouraged from browsing the internet for solutions to problems.

3. Please ask questions of the teacher in class.

4. Please don't talk in class, and please put your cell phones away. But, once again, *do* ask questions. If something seems unclear please let us know.

Preliminary Problem Set

Solutions are due on 10 May, at the beginning of class. This will not count toward your course grade. Roger and Sylvia will make comments on papers and return them to students on 11 May (maybe).

1. If G is a group with identity element 1 such that $x^2 = 1$ for every $x \in G$, prove that G is abelian.

2. If R is a ring such that $x^2 = x$ for every $x \in R$, prove
(a) $2x = 0$ for every $x \in R$, and
(b) R is commutative.

3. Factor the polynomial $x^3 + 2x - 1$ over the field F completely (into a product of irreducible polynomials that you justify are irreducible).
(a) If $F = \mathbb{Q}$, the rational numbers.
(b) If $F = \mathbb{C}$, the complex numbers.
(c) If $F = \mathcal{F}_5 = \mathbb{Z}/5\mathbb{Z}$, the field of integers mod 5.