Tribhuvan University

Institute of Science and Technology

Kirtipur, Kathmandu Nepal

Final Examination 2074 September

Subject: **Mathematics (Field and Galois theory)**	*Full Marks*: 60
Course No.: Math 724	*Pass Marks*: 30
Level: **M. Phil.(math)**/I Semester	*Time*: 2:00 hr

Attempt ***any 5 questions****. Each question carries equal marks. Write your answer in detail as far as possible.*

1. Show that $\cos(2\pi/9)$ is algebraic and determine its minimal polynomial over $\mathbb{Q}$.
2. Let α be an algebraic complex number. Give the definition of the minimal polynomial f_α of α over $\mathbb{Q}$ and prove that $\deg f_\alpha = [\mathbb{Q}[\alpha] : \mathbb{Q}]$.
3. Show that $\mathbb{Q}[\sqrt{5} - \sqrt{6}] = \mathbb{Q}[\sqrt{5}, \sqrt{6}]$, compute the dimension $[\mathbb{Q}[\sqrt{5} - \sqrt{6}] : \mathbb{Q}]$ and determine all the subfields of $\mathbb{Q}[\sqrt{5} - \sqrt{6}p]$ justifying your answers.
4. Show that any algebraic extension of fields is necessarily finite.
5. Let $F_1 = \mathbb{F}[\gamma], \gamma^3 = \gamma + 1$ and $F_2 = \mathbb{F}[\delta], \delta^3 = \delta^2 + 1$. Determine $|F_1|$, $|F_2|$ and show that $F_1 \cong F_2$ by describing an explicit isomorphism between them.
6. Describe the splitting field of the polynomial $(X^4 - 3)(X^2 - 2)$ and write down the elements of its Galois group.
7. State in its full generality the *Fundamental Theorem of Galois Theory* (NOTE: sometimes it is also called the *Galois Correspondance Theorem*).