Tribhuvan University
Institute of Science and Technology
Kirtipur, Kathmandu Nepal

Final Examination 2074 September

Subject: **Mathematics (Field and Galois theory)** *Full Marks*: 60
Course No.: Math 724 *Pass Marks*: 30
Level: **M. Phil.(math)**/I Semester *Time*: 2:00 hr

Attempt ***any 5 questions****. Each question carries equal marks. Write your answer in detail as far as possible.*

1. Show that $\cos(2\pi/7)$ is algebraic and determine its minimal polynomial over $\mathbb{Q}$.
2. Let α be an algebraic complex number. Give the definition of the minimal polynomial f_α of α over $\mathbb{Q}$ and prove that $\deg f_\alpha = [\mathbb{Q}[\alpha] : \mathbb{Q}]$.
3. Show that $\mathbb{Q}[\sqrt{3}-\sqrt{2}] = \mathbb{Q}[\sqrt{3}+\sqrt{2}]$, compute the dimension $[\mathbb{Q}[\sqrt{3}+\sqrt{2}] : \mathbb{Q}]$ and determine all the subfields of $\mathbb{Q}[\sqrt{3}-\sqrt{2}]$ justifying your answers.
4. Show that any algebraic extension of fields is necessarily finite.
5. Let $F_1 = \mathbb{F}[\alpha], \alpha^3 = \alpha + 1$ and $F_2 = \mathbb{F}[\beta], \beta^3 = \beta^2 + 1$. Determine $|F_1|$, $|F_2|$ and show that $F_1 \cong F_2$ by describing an explicit isomorphism between them.
6. Describe the splitting field of the polynomial $(X^4 - 2)(X^2 - 3)$ and write down the elements of its Galois group.
7. State in its full generality the *Fundamental Theorem of Galois Theory*
(NOTE: sometimes it is also called the *Galois Correspondance Theorem*).