These excercises are due July 12, 2017, at 10 pm. Nepal time. Please, send them to nap@rnta.eu and to laurageatti@gmail.com and schoof.rene@gmail.com. Contact us if you have any question!

1. Let p be a prime. Prove that there are infinitely many irreducible polynomials in $\mathbf{F}_p[x]$.
2. (a) Prove that the Frobenius automorphism of $\mathbf{F}_5[\sqrt{2}]$ sends $\sqrt{2}$ to $-\sqrt{2}$.
   (b) Compute the order of $1-\sqrt{2},\ 2-\sqrt{2},\ 3-\sqrt{2}$ in $\mathbf{F}_5[\sqrt{2}]^*$.
3. How many proper subfields does $\mathbf{F}_{2^{12}}$ have? Explain...
4. Is the polynomial x^2+x+1 irreducible or not in $\mathbf{F}_2[x]$? and in $\mathbf{F}_4[x]$?
5. Given the polynomial x^3+2 in $\mathbf{F}_5[x]$, compute the order of its roots in the multipicative group of its splitting field.
6. Give an explicit isomorphism $\mathbf{F}_5[x]/(x^2+x+1) \to \mathbf{F}_5[\sqrt{2}]$.
7. (a) What is the degree of the smallest field extension of $\mathbf{F}_5$ which contains an element of multiplicative order 13.
   (b) Determine the degrees of the irreducible factors of $x^{13}-1$ in $\mathbf{F}_5[x]$.
8. Compute the discriminant of the polynomial $x^7+x+1 \in \mathbf{Z}[x]$.
   Note: Question number 8 is optional.
9. Let $f = x^2+x+1$ in $\mathbf{F}_2[x]$. Show that its Galois group G_f is not contained in $A_2 = \{Id\}$, despite the fact that its discriminant is a square in $\mathbf{F}_2$.
10. Exhibit a transitive subgroup of S_4 different from A_4 and S_4.