2017 NAP Lecture Module III, Problem 3

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May 31, 2017

To solve the following problems, we assume all arguments up to Prop. 3.20 and those in Module III.

Problem 3] (From Chapter Exercise 1-4).

3-1] Let $a \in \boldsymbol{R}^+$ be a given number. Construct $\sqrt{a}$ (> 0) by ruler and compass.

vskip 3mm 3-2] (1) We assume that we can describe the trisector of any given constructible angle. Interprete this assumption to the constructibility of the solution of the cubic equation $4x^3 - 3x - k = 0$ for any given constructible number k.

(2) Under the above assumption (1), show that we can construct the regular 7-gon.

3-3] (1) Describe the cubic equation for $\cos(\frac{2\pi}{7})$.

(2) Set $\zeta = e^{2\pi i/7}$, and set $\beta = \zeta + \zeta^2 + \zeta^4, \beta' = \overline{\beta} = \zeta^3 + \zeta^5 + \zeta^6$. Show that $(\beta - \beta')^2 = -7$. (By this calculation, we know $\boldsymbol{Q}[\beta] = \boldsymbol{Q}[\beta'] = \boldsymbol{Q}[\sqrt{-7}]$.)

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