2017 NAP Lecture Module III, Problem 1

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Restatement of Remark 3.11 (b) This remark in the text book does not make sense. Prof.s Wiegands noticed it and omitted from their exposition. But it is used many times in later sections. So I (=HS) put here the corrected version for the later use.

(b) Let E/F be a separable finite extension, and let G be a finite group of $\mathrm{Aut}(E/F)$. Then Prop. 2.7 (a) says $\sharp\mathrm{Aut}(E/E^G) = [E : E^G]$. On the other hand, by (3.5), it holds $\mathrm{Gal}(E/E^G) = G = \mathrm{Aut}(E/E^G)$. So we have $\sharp G = [E : E^G]$.

(b bis) Especially, in case E/F is Galois (with the Galois group G), we have

(i) $(G =)\mathrm{Gal}(E/F) = \mathrm{Aut}(E/F)$, (ii) $\sharp\mathrm{Gal}(E/F) = [E : F]$.

·) We wish to recall several fundamental facts among what we have already studied in the previous Modules.

··)We assume all the results studied in the Modules I, II (i.e. up to Thm 3.16 in the summary).

Problem 1-1] Set $f(X) = X^4 + X + 1, P(X) = X^3 - X + 1 \in \boldsymbol{Q}[X]$. Show the procedure to obtain $\gcd(f, P)$ by the Euclidean algorism.

Problem 1-2]. Let α be a solution of $f(X) = 0$ and set $E = \boldsymbol{Q}[\alpha]$. Show the inverse of $P(\alpha)$ in E, provided $P(\alpha) \neq 0$.

Problem 1-3]. Let F be a field and let $f(X)$ be an irreducible polynomial in $F[X]$. Let α be a root of $f(X)$. By using the Euclidean method, show that any $Q[\alpha] \in F[\alpha] - \{0\}$ has its inverse in $F[\alpha]$.

(This means that $F(\alpha) = F[\alpha]$. So, in this case, we write $F[\alpha]$ instead of the field $F(\alpha)$ in the later sections).

Problem 1-4]. Show that for any prime p, $X^3 - p$ is irreducible in $\boldsymbol{Q}[X]$.

Problem 1-5]. Let α be a real root of $X^3 - p = 0$. Show that $\boldsymbol{Q}[\alpha]/\boldsymbol{Q}$ is not a Galois extension.

NAP Lecture Part II Exercises Sheet

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[Solutions]