NEPAL ALGEBRA PROJECT 2017
MODULE 2 — PROBLEM SET #1

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16 May, 2017

1. Milne, Exercise 1-1 (p. 25)

2. Milne, Exercise 1-2 (p. 25)

3. Milne, Exercise 1-5 (p. 25).

4. Milne, Exercise 2-1 (p. 33)

5. Milne, Exercise 2.2 (p. 33)

6. Milne, Exercise 2.3 (p.33)

7. Milne, Exercise 2.6 (p.33)

8. Find the splitting field, and its degree over $\mathbb{Q}$, for the polynomial $f = X^4 + X^2 + 1 \in \mathbb{Q}[X]$. (Hint: Think about $(X^2 - 1)f(X)$.)

9. Find an irreducible polynomial $f \in \mathbb{Q}[X]$, with roots $\alpha_1, \alpha_2, \alpha_3, \alpha_4 \in \mathbb{C}$ such that

$$[\mathbb{Q}(\alpha_1, \alpha_2) : \mathbb{Q}] \neq [\mathbb{Q}(\alpha_3, \alpha_4) : \mathbb{Q}].$$

(In particular, the fields $\mathbb{Q}(\alpha_1, \alpha_2)$ and $\mathbb{Q}(\alpha_3, \alpha_4)$ are not isomorphic. The point of this exercise is to clarify the last sentence of Remark 2.9 (b), in Milne (on p. 30): We know that $\mathbb{Q}(\alpha_i) \cong \mathbb{Q}(\alpha_j)$ for all i, j, but adjoining *two* roots is very different.)

These problems are due Tuesday, 23 May, 2017, at 10 pm Nepal time. They must be sent to

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(copy to rwiegand1@unl.edu and swiegand1@unl.edu) by 10 pm Nepal time.

You may discuss problems with other students in the class, but you must do the write-up completely by yourself, without consulting anyone else. You may refer, by number, to theorems, propositions, etc., in Milne's book, provided that they are results that were covered in NAP Module 1 or in the first three meetings of Module 2: 16 May – 18 May). If you are ambitious, you can write your solutions in TeX and send them as an attachment. Alternatively, you can write them out (legibly, please), scan them, and send as an attachment. A less desirable option would be to photograph your solutions and send the photo; this will probably be harder to read, so the first two options are preferable.

You can download Milne's book at http://www.jmilne.org/math/CourseNotes/FT.pdf

The NAP website is: http://www.rnta.eu/nap/

Feel free to email us anytime with questions about the course or the homework, or about other mathematical issues!