Nepal Algebra Project(NAP)
Central Department of Mathematics
Tribhuvan University, Kirtipur, Kathmandu, Nepal
Fields and Galois Theory- Short Note of Module 4 - Lecture 6
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The Galois group of a polynomial

- Construction of an irreducible polynomial of degree p, with Galois group $G_f = S_p$, for every prime p.
- A method to compute Galois groups $Gal(\mathbf{K}_f/\mathbf{Q})$. By reducing f modulo various primes p, one computes automorphisms σ_p in the Galois group $G_f \subset S_n$ up to conjugacy in G_f. The cycle decomposition of σ_p is determined by the factorization of the polynomial f modulo p. This method can be used to show that the Galois group G_f is "large". To show that it is "small", other method are needed (see Milne, p.55). The method was illustrated by a computer presentation using ***PARI/GP***.

Excercises.

1. Let $\mathbf{K}$ be a field of $char \neq 2$. If $u, v \in \mathbf{K}^*$, then $\mathbf{K}(\sqrt{u}) = \mathbf{K}(\sqrt{v})$ if and only if u/v is a square in $\mathbf{K}^*$.
2. Compute the Galois group of $\mathbf{Q}_f$, where $f(x) = X^4 + 5x + 5$ (irreducible over $\mathbf{Q}$).
3. Fix $\alpha = \sqrt{2 - \sqrt{3}}$.
   (a) Show that $\mathbf{Q}(\alpha)$ is a normal extension of $\mathbf{Q}$.
   (b) Compute its Galois group.
4. Let f be the polynomial $x^4 - 2$. Compute the Galois group of $\mathbf{K}_f$, when $\mathbf{K} = \mathbf{Q}, \mathbf{R}, \mathbf{C}, \mathbf{Q}(\sqrt{2}), \mathbf{Q}(\sqrt[4]{2})$.