Nepal Algebra Project(NAP)
Central Department of Mathematics
Tribhuvan University, Kirtipur, Kathmandu,Nepal
Fields and Galois Theory- Short Note of Module 4 - Lecture 5
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The Galois group of a polynomial

$f \in \mathbf{K}[x]$, separable monic polynomial of *degree 4*;
g the cubic resolvent of f.
f separable $\Rightarrow g$ separable;
$disc(f) = disc(g)$.

Proposition. $\mathbf{K}[\alpha, \beta, \gamma]$ is the fixed field of $G_f \cap V_4$.

Classification of the Galois groups of irreducible separable monic polynomials of *degree* 4, by means of the cubic resolvent:

1. g irred., $disc(g)$ not square, $G_g = S_3$, then $G_f = S_4$;
2. g irred., $disc(g)$ square, $G_g = A_3$, then $G_f = A_4$;
3. $g = h_1 \cdot h_2$, with $deg(h_i) = i$, $disc(g)$ not square, $G_g = C_2$, then $G_f = D_4 \Leftrightarrow f$ irred. in $\mathbf{K}_g$;
4. $g = h_1 \cdot h_2$, with $deg(h_i) = i$, $disc(g)$ not square, $G_g = C_2$, then $G_f = C_4 \Leftrightarrow f$ red. in $\mathbf{K}_g$;
5. g completely red., $disc(g)$ not square, $G_g = id$, then $G_f = V_4$.

Construction of an irreducible polynomial of degree p, with Galois group $G_f = S_p$, for every prime p.

Lemma. Let H be a subgroup of S_p. If H contains a 2-cycle and a p-cycle, then $H = S_p$.

We will construct a polynomial $f \in \mathbf{Q}[x]$, irreducible of degree p, with $p-2$ real roots and 2 complex conjugate roots.

Excercises. Computation of the Galois group of the following irreducible polynomials of degree 4:

1. $x^4 - x - 1$ (type S_4);
2. $x^4 + 2x + 2$ (type S_4);
3. $x^4 + 8x + 12$ (type A_4);
4. $x^4 + 36x + 63$ (type V_4);
5. $x^4 - 10x^2 + 5$ (type C_4);
6. $x^4 + 3x + 3$ (type D_4);