

Tabella 1. The field  $\mathbf{F}_{16}$ .

$\alpha^i$	additive	minimum polynomial	order in $\mathbf{F}_{16}^*$
—	0	$X$	—
1	1	$X + 1$	1
$\alpha$	$\alpha$	$X^4 + X + 1$	15
$\alpha^2$	$\alpha^2$	$X^4 + X + 1$	15
$\alpha^3$	$\alpha^3$	$X^4 + X^3 + X^2 + X + 1$	5
$\alpha^4$	$\alpha + 1$	$X^4 + X + 1$	15
$\alpha^5$	$\alpha^2 + \alpha$	$X^2 + X + 1$	3
$\alpha^6$	$\alpha^3 + \alpha^2$	$X^4 + X^3 + X^2 + X + 1$	5
$\alpha^7$	$\alpha^3 + \alpha + 1$	$X^4 + X^3 + 1$	15
$\alpha^8 = \alpha^{-7}$	$\alpha^2 + 1$	$X^4 + X + 1$	15
$\alpha^9 = \alpha^{-6}$	$\alpha^3 + \alpha$	$X^4 + X^3 + X^2 + X + 1$	5
$\alpha^{10} = \alpha^{-5}$	$\alpha^2 + \alpha + 1$	$X^2 + X + 1$	3
$\alpha^{11} = \alpha^{-4}$	$\alpha^3 + \alpha^2 + \alpha$	$X^4 + X^3 + 1$	15
$\alpha^{12} = \alpha^{-3}$	$\alpha^3 + \alpha^2 + \alpha + 1$	$X^4 + X^3 + X^2 + X + 1$	5
$\alpha^{13} = \alpha^{-2}$	$\alpha^3 + \alpha^2 + 1$	$X^4 + X^3 + 1$	15
$\alpha^{14} = \alpha^{-1}$	$\alpha^3 + 1$	$X^4 + X^3 + 1$	15

Tabella 2. The field  $\mathbf{F}_{27}$ .

$\beta^i$	additive	minimum polynomial	order in $\mathbf{F}_{27}^*$
—	0	$X$	—
1	1	$X - 1$	1
$\beta$	$\beta$	$X^3 - X + 1$	26
$\beta^2$	$\beta^2$	$X^3 + X^2 + X - 1$	13
$\beta^3$	$\beta - 1$	$X^3 - X + 1$	26
$\beta^4$	$\beta^2 - \beta$	$X^3 + X^2 - 1$	13
$\beta^5$	$-\beta^2 + \beta - 1$	$X^3 - X^2 + X + 1$	26
$\beta^6$	$\beta^2 + \beta + 1$	$X^3 + X^2 + X - 1$	13
$\beta^7 = -\beta^{-6}$	$\beta^2 - \beta - 1$	$X^3 + X^2 - X + 1$	26
$\beta^8 = -\beta^{-5}$	$-\beta^2 - 1$	$X^3 - X^2 - X - 1$	13
$\beta^9 = -\beta^{-4}$	$\beta + 1$	$X^3 - X + 1$	26
$\beta^{10} = -\beta^{-3}$	$\beta^2 + \beta$	$X^3 + X^2 - 1$	13
$\beta^{11} = -\beta^{-2}$	$\beta^2 + \beta - 1$	$X^3 + X^2 - X + 1$	26
$\beta^{12} = -\beta^{-1}$	$\beta^2 - 1$	$X^3 + X^2 - 1$	13
$\beta^{13} = -1$	-1	$X + 1$	2
$\beta^{14} = -\beta$	$-\beta$	$X^3 - X - 1$	13
$\beta^{15} = -\beta^2$	$-\beta^2$	$X^3 - X^2 + X + 1$	26
$\beta^{16} = -\beta^3$	$-\beta + 1$	$X^3 - X - 1$	13
$\beta^{17} = -\beta^4$	$-\beta^2 + \beta$	$X^3 - X^2 + 1$	26
$\beta^{18} = -\beta^5$	$\beta^2 - \beta + 1$	$X^3 + X^2 + X - 1$	13
$\beta^{19} = -\beta^6$	$-\beta^2 - \beta - 1$	$X^3 - X^2 + X + 1$	26
$\beta^{20} = \beta^{-6}$	$-\beta^2 + \beta + 1$	$X^3 - X^2 - X - 1$	13
$\beta^{21} = \beta^{-5}$	$\beta^2 + 1$	$X^3 + X^2 - X + 1$	26
$\beta^{22} = \beta^{-4}$	$-\beta - 1$	$X^3 - X - 1$	13
$\beta^{23} = \beta^{-3}$	$-\beta^2 - \beta$	$X^3 - X^2 + 1$	26
$\beta^{24} = \beta^{-2}$	$-\beta^2 - \beta + 1$	$X^3 - X^2 - X - 1$	13
$\beta^{25} = \beta^{-1}$	$-\beta^2 + 1$	$X^3 - X^2 + 1$	26

NB. Since  $X^3 - X + 1 = (X - \beta)(X - \beta^3)(X - \beta^9)$ , we have  $\beta^{13} = \beta\beta^3\beta^9 = -1$ .