Nepal Algebra Project(NAP)
Fields and Galois Theory “multiple hands" course in Nepal
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NAP: Module -2, Lecture -6, Thursday, 25 May, 2017

- First, there was a typo in the notes on Class #5: Twelve lines from the bottom, “$\mathbb{Q}(\sqrt[3]{2}) \supset \mathbb{Q}(\sqrt[3]{2}) \supset \mathbb{Q}$” should have been “$\mathbb{Q}(\sqrt[3]{2}, \omega) \supset \mathbb{Q}(\sqrt[3]{2}) \supset \mathbb{Q}$”.
- Probably there should be another equivalent condition for “Galois” in Theorem 3.10, namely, (e) $|\operatorname{Aut}(E/F)| = [E : F] < \infty$. To see that this is equivalent, note that $\operatorname{Aut}(E/F) = \operatorname{Aut}(E/E^G)$, where $G = \operatorname{Aut}(E/F)$. Since $|G| = [E : E^G]$ (the important point that *should* have been part of Corollary 3.5), we see that (e) is equivalent to (d).
- In Corollary 3.12 there's the same problem encountered in Theorem 3.10: You have to take the *distinct* f_i.
- I skipped Remark 3.14 because I wanted to be sure to have time for FTGT.
- Neither Sylvia nor I could make any sense out of Remark 3.11 (b), so we told the students to ignore it. Unfortunately the proof of FTGT refers to 3.11 (b) twice, but actually Corollary 3.5 is what is needed. I added a little to FTGT, namely, that the intermediate field M is normal (Galois) over F if and only if it's stable under the action of the Galois group. (Of course, this is what Milne proves, but it seems worthwhile to make it explicit.)
- I drew lots of side-by-side pictures of field extensions and upside down subgroup lattices, and explained how joins correspond to intersections. (Reminded that for subgroups H_1 and H_2 the product H_1H_2 is not necessarily a subgroup, so I used “$\langle H_1H_2\rangle$” for the join. I did not get to 3.18 or 3.19 or 3.20.
- I illustrated FTGT with splitting fields of $(X^2 - 2)(X^2 - 3)$ and $X^3 - 2$ over $\mathbb{Q}$ (with side-by-side pictures of intermediate fields and subgroups). I told them to go over Example 3.21 and draw these pictures.