

Nepal Algebra Project 2016 Final exam – VERSION **B**

Tribhuvan University

July 30th 2016

1. Consider the polynomial

$$f(X) = X^5 - 9X \in \mathbb{Z}[X].$$

Let E be the decomposition field of f over \mathbb{Q} .

- (a) Give a basis of E over \mathbb{Q} .
(6 marks)
- (b) Check that the Galois group G_f of f over \mathbb{Q} is isomorphic to the subgroup of \mathfrak{S}_5 generated by two disjoint transpositions.
(6 marks)
- (c) For each subgroup H of G_f , give the subfield E^H of E fixed by H .
(6 marks)
- (d) Give the list of subfields of E .
(6 marks)
- (e) Give a primitive element of E over \mathbb{Q} .
(6 marks)

2. Let $F = \mathbb{Q}(\sqrt{5}, \sqrt[3]{5})$.

- (a) Find $[F : \mathbb{Q}]$.
(6 marks)
- (b) Is F normal over \mathbb{Q} ?
(6 marks)
- (c) Give a primitive element of F over \mathbb{Q} .
(6 marks)

3. Find the order of the Galois group of $x^5 - 3$.

(6 marks)

4. State the fundamental Theorem of Galois Theory (Galois correspondance).

(6 marks)