Nepal Algebra Project 2016 Final exam

Tribhuvan University

July 30^{th} 2016

1. Consider the polynomial
$$f(X) = X^5 - 4X \in \mathbb{Z}[X].$$
Let E be the decomposition field of f over $\mathbb{Q}$.

   (a) Give a basis of E over $\mathbb{Q}$.

   (6 marks)

   (b) Check that the Galois group G_f of f over $\mathbb{Q}$ is isomorphic to the subgroup of $\mathfrak{S}_5$ generated by two disjoint transpositions.

   (6 marks)

   (c) For each subgroup H of G_f, give the subfield E^H of E fixed by H.

   (6 marks)

   (d) Give the list of subfields of E.

   (6 marks)

   (e) Give a primitive element of E over $\mathbb{Q}$.

   (6 marks)

2. Let $F = \mathbb{Q}(\sqrt{2}, \sqrt[3]{2})$.

   (a) Find $[F : \mathbb{Q}]$.

   (6 marks)

   (b) Is F normal over $\mathbb{Q}$?

   (6 marks)

   (c) Give a primitive element of F over $\mathbb{Q}$.

   (6 marks)

3. Find the order of the Galois group of $x^5 - 2$.

   (6 marks)

4. State the fundamental Theorem of Galois Theory (Galois correspondance).

   (6 marks)