Nepal Algebra Project(NAP)
Central Department of Mathematics
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Fields and Galois Theory

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NAP: Module-4, Problem Set 1

1. *The symmetric group* $\mathfrak{S}_4$.

(a) Check that, among the 24 elements of the symetric group $\mathfrak{S}_4$,

- 1 has order 1
- 9 have order 2
- 8 have order 3
- 6 have order 4

`Hint:` the partitions of 4 are (1)(1)(1)(1), (2)(1)(1), (2)(2), (3)(1), (4).

(b) Deduce that in $\mathfrak{S}_4$ there are 30 subgroups:

- 1 with order 1
- 9 with order 2
- 4 with order 3
- 7 with order 4
- 4 with order 6
- 3 with order 8
- 1 with order 12
- 1 with order 24

(c) Check that there are 11 conjugacy classes and 4 normal subgroups.

`Reference:`
`http://groupprops.subwiki.org/wiki/Subgroup_structure_of_symmetric_group:S4`

2. *The dihedral group* D_n *of order* n.

(a) Let $n \geq 1$. Consider the following two elements r and s in $\mathfrak{S}_n$:

$$r(i) = i + 1 \mod n, \quad s(i) = n + 2 - i \mod n \qquad (i = 1, 2, \ldots, n).$$

Check that $r^n = 1$, $s^2 = 1$, $rsrs = 1$ and that D_n is a subgroup of $\mathfrak{S}_n$ of order $2n$.
This is the dihedral group of index n, group of symmetries of the regular n–gone.
(b) Show that if n is odd, then D_{2n} is isomorphic to the direct product $C_2 \times D_n$.
(c) Give the list of groups of order $2p$ with p prime.
`Hint.` Use the fact that such a group contains an element of order p and an element of order 2.

3. *A transitive subgroup of* $\mathfrak{S}_n$ *containing a* $n-1$ *cycle and a transposition is* $\mathfrak{S}_n$.

(a) Let $\sigma = (1, 2, \cdots, n-1)$ and $\tau = (1, n)$. Check

$$\sigma\tau\sigma^{-1} = (2, n).$$

(b) Check that $\mathfrak{S}_n$ is generated by τ and σ.
(c) Let G be a transitive subgroup of $\mathfrak{S}_n$ containing a $n-1$ cycle and a transposition. Check $G = \mathfrak{S}_n$.