NEPAL ALGEBRA PROJECT 2016

MODULE III MODEL QUESTIONS

(1) Let $f = X^3 - 2$ over $\mathbb{Q}$, and let $\omega = e^{2\pi i/3}$.
   (a) Prove that $E = \mathbb{Q}[\sqrt[3]{2}, \omega]$ is a splitting field for f.

   (2 marks)

   (b) Calculate $|E : \mathbb{Q}|$ and give the structure of $\mathrm{Gal}(E/\mathbb{Q})$.

   (4 marks)

   (c) Write down the lattice of intermediate subfields for the extension $E/\mathbb{Q}$.

   (4 marks)

(2) (a) Give the definition of a constructible angle.

   (2 marks)

   (b) Prove that the angle $\dfrac{2\pi}{34}$ is constructible.

   (3 marks)

   (c) Explain how to construct the number $\sqrt{7}$ using ruler and compass, and prove that the number $\sqrt[3]{7}$ is not constructible.

   (5 marks)