2016 NAP Lecture Part II, Problem 2

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To solve the following problems, we assume all arguments in Chapter 1 and Chapter 2.

Problem II-4] Let F be a finite field with char(F) = p(> 0). Show that $F = \{\text{roots of the equation } X^{p^n} - X = 0\}$, where $n = [F : \mathbf{F}_p]$.

(So, F is determined uniquely, once p and n are given.)

Problem II-5] From excersises in Chap. 1.

(1) Let $a \in \mathbf{R}^+$ be a given number. Construct \sqrt{a} (> 0).

(2) We assume that we can describe the trisector of any given angle. Interpret this assumption to the sovability of a certain cubic equation.

(3) Under the above assumption, show that we can describe a regular 7-gon.

Problem II-6] From excersises in Chap. 1.

Let F be a field, and let $f(X) \in F[X]$ be an irreducible polynomial with $\deg(f) = n$. Suppose E be an extension of F with [E:F] = m. Assume $\gcd(m,n) = 1$. Show that f(X) is irreducible in E[X], also.

NAP Lecture Part II Exercises Sheet

Given Name

Family Name

Date: d /m /2016

Status

Specialized Field