2016 NAP Lecture Part II, Problem 1

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To solve the following problems, we assume all arguments in Chapter 1.

Problem II-1] Set $X = \{a \in \boldsymbol{C} :\ \text{a is an algebraic number}\}$. By using Prop. 1.44, show that X is an algebraic closure of $\boldsymbol{Q}$. (We denote X by $\overline{\boldsymbol{Q}}$.)

Problem II-2] Let F be a field of characteristic $p(> 0)$. Suppose $a \in F$ is not a p-th power in F (i.e. We don't have $a = \alpha^p$ for any $\alpha \in F$). Show that $f(X) = X^p - a$ is irreducible in $F[X]$.

Problem II-3. From Exercises of Chap 1.

(1) Let G be a finite abelian group with $\sharp(G) = n$. Assume the property :

(d)If for every divisor d of $n = \sharp(G)$ the number of solutions of $x^d - 1 = 0$ does not exceed d, then G is a cyclic group.

Show that the multiplicative group F^* of a finte field F is a cyclic group.

(2) Problem II-3 bis. Prove the above property (d).