# NAP PROBLEM SET #1

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1. Use the Euclidean algorithm to find the greatest common divisor $d(X)$ of the polynomials

$$X^3 + X^2 + 2X + 2 \quad \text{and} \quad X^5 + X + 1 \quad \text{in } \mathbb{F}_3[X]\,,$$

and find polynomials $f(X), g(X) \in \mathbb{F}_3[X]$ such that

$$f(X)(X^3 + X^2 + 2X + 2) + g(X)(X^5 + X + 1) = d(X)\,.$$

2. Prove that $X^4 - 10X^2 + 1$ is irreducible in $\mathbb{Q}[X]$.

3. Let $R$ and $S$ be commutative rings, let $\varphi : R \to S$ be a ring homomorphism, and let $\alpha$ be an arbitrary element of $S$. Prove that $\varphi$ extends *uniquely* to a ring homomorphism $\hat{\varphi} : R[X] \to S$ such that $\hat{\varphi}(c) = \varphi(c)$ for $c \in R$ and $\hat{\varphi}(X) = \alpha$. (Note: This is part of 1.5 in Milne, and it says that "substitution" behaves well with respect to addition and multiplication, a fact that we used in the first lecture.)

4. Let $G$ be a finite cyclic group of even order. We say that an element $g \in G$ is a *square* provided there is an element $\ell \in G$ such that $\ell^2 = g$. Suppose $g$ and $h$ are elements of $G$ and neither one of them is a square. Prove that $gh$ *is* a square.

5. For each of the following polynomials in $\mathbb{Q}[X]$, either prove that it is irreducible in $\mathbb{Q}[X]$, or factor it into irreducible factors in $\mathbb{Q}[X]$ and prove that each of these factors is irreducible:

a) $X^4 + X^2 + 1$
b) $X^4 + 1$
c) $X^5 - 1$
d) $X^9 + X^3 + 1$
e) $X^4 + X^3 + X^2 + X + 1$ (Hint: Let $X = Y + 1$.)

These problems are due Sunday, 15 May, 2016. They must be sent to

nap@rnta.eu

(copy to rwiegand1@unl.edu and swiegand1@unl.edu) by 10 pm Nepal time.

You may discuss problems with other students in the class, but you must do the write-up completely by yourself, without consulting anyone else. You may refer, by number, to theorems, propositions, etc., in Milne's book, provided that they are results that we have covered in the

the first three meetings of the class (8, 10, 11 May, 2016). If you are ambitious, you can write your solutions in TeX and send them as an attachement. Alternatively, you can write them out (legibly, please), scan them, and send as an attachment. A less desirable option would be to photograph your solutions and send the photo; this will probably be harder to read, so the first two options are preferable.

You can download Milne's book at http://www.jmilne.org/math/CourseNotes/FT.pdf

The NAP website is: http://www.rnta.eu/nap/

Feel free to email us anytime with questions about the course or the homework, or about other mathematical issues!