Nepal Algebra Project(NAP)
Central Department of Mathematics
Tribhuvan University,Kirtipur,
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Fields and Galois Theory

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Summary of NAP:Module 4-Lecture 4

- The symmetric group $\mathfrak{S}_n$ and the alternating group $\mathfrak{A}_n$.The symmetric group $\mathfrak{S}_n$. Examples: $n = 1, 2, 3, 4$. Order. Transpositions, cycles. Decomposition into disjoint cycles. Relation

$$(a_1, a_2)(a_2, a_3) \cdots (a_{k-1}, a_k) = (a_1, a_2, \ldots, a_k).$$

- Inversion, number of inversions, signature: sign $= (-1)^{\#\text{inversions}}$.

- Group acting on a set: definition. Any group G is isomorphic to a subgroup of the symmetric group $\mathfrak{S}_G$. In particular any finite group of order n is isomorphic to a subgroup of $\mathfrak{S}_n$.

- Action of $\mathfrak{S}_n$ on the set of maps $\mathbf{Z}^n \to \mathbf{Z}$

$$\sigma f(a_1, \ldots, a_n) = f(a_{\sigma(1)}, \ldots, a_{\sigma(n)}).$$

  Example:

$$p(a_1, \ldots, a_n) = \prod_{1 \le i \le n} (a_j - a_i).$$

- Consequence: $\sigma : \mathfrak{S}_n \to \{-1, +1\}$ is a group homomorphism, surjective for $n \ge 2$. Kernel: alternating group $\mathfrak{A}_n$.

- Signature of a cycle of length k: $(-1)^{k-1}$.
- Discriminant of a polynomial. Proposition 4.1 and Corollary 4.2 of Milne. Polynomials of degree 2 and 3: examples 4.5 and 4.6.