

Nepal Algebra Project(NAP)
Central Department of Mathematics
Tribhuvan University,Kirtipur,
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Fields and Galois Theory

Course Instructor: Prof. Michel Waldschmidt

Lecture-3

- Examples of normal and of non normal extensions. Stem field, splitting field.
- Quadratic extensions. Galois group when separable.
- Galois group of a polynomial over a field as a subgroup of \mathfrak{S}_n .
Example: the two polynomials $f_1 = X^4 - 8X^2 + 15$ and $f_2(X) = X^4 - 16X^2 + 4$ have the same splitting field over \mathbf{Q} , since

$$f_1 = (X^2 - 3)(X^2 - 5)$$

and

$$f_2 = (X - \sqrt{5} - \sqrt{3})(X - \sqrt{5} + \sqrt{3})(X + \sqrt{5} - \sqrt{3})(X + \sqrt{5} + \sqrt{3});$$

however, as subgroups of \mathfrak{S}_4 , the Galois group of f_1 over \mathbf{Q} is transitive but not the Galois group of f_2 .