Nepal Algebra Project(NAP)
Central Department of Mathematics
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Fields and Galois Theory

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Summary of NAP: Module 3, Lecture 4

1. Defined a constructible point using the definition given in Garling.
2. We then formally defined a constructible number to be a number c such that the point $(c, 0)$ is constructible. (A formal definition is not given in Milne.)
3. Writing $\mathscr{C}$ for the set of all constructible numbers, we proved that
   (a) the set of all constructible points is $\mathscr{C} \times \mathscr{C}$;
   (b) $\mathscr{C}$ is a field that contains $\mathbb{Q}$;
   (c) Theorem 1.6 of Milne: a real number z is constructible if and only if it is contained in a field of form $\mathbb{Q}[\sqrt{a_1}, \sqrt{a_2}, \dots, \sqrt{a_k}]$ where $a_i > 0$ and $a_i \in \mathbb{Q}[\sqrt{a_1}, \sqrt{a_2}, \dots, \sqrt{a_i}]$;
   (d) if z is construcible, then z lies in an extension E such that $|E : \mathbb{Q}| = 2^k$. In particular $\mathscr{C}$ is contained in the set of all algebraic numbers but, on the other hand, $\sqrt[3]{2}$ is not constructible.