## Nepal Algebra Project(NAP) Central Department of Mathematics Tribhuvan University, Kirtipur, Kathmandu, Nepal Fields and Galois Theory

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## Summary of NAP: Module -1, Lecture 5, On Tuesday May 17 2016

Discussed extension fields, notation, as on page 13 of Milne, including "F-homomorphism", and examples. We added the fact, which we proved later, that, if  $\alpha$  and  $\beta$  are roots of the same irreducible polynomial there exists an F-homomorphism from  $F[\alpha]$  to  $F[\beta]$ 

Proved Proposition 1.20 Multiplication of degrees, part 2. (Will go over part 1 in lecture 4 today.)

Proved Lemma 1.2.3: For  $F \subseteq R$ , where F is a field and R is an integral domain that has finite dimension over F as a vector space, R is a field.

Discussed constructing an extension field where an irreducible polynomial has a root. Proved it is a field.

Discussed examples such as building the complex numbers from the reals.

Proved the finiteness Theorem: For  $F \subseteq E \subseteq K$  fields, these are equivalent:

- 1. E/F is finite (vector space dimension).
- 2. E/F is finitely generated as a field extension and is algebraic over F.
- 3. There exist algebraic elements  $\alpha_1, \alpha_2, \ldots, \alpha_n$  of E, such that  $E = F(\alpha_1, \alpha_2, \ldots, \alpha_n)$ .