Tribhuvan University

Institute of Science and Technology

Kirtipur, Kathmandu Nepal

Final Examination 2073 September

Subject: **Mathematics (Field and Galois theory)** *Full Marks*: 60
Course No.: Math 724 *Pass Marks*: 30
Level: **M. Phil.(math)**/I Semester *Time*: 2:00 hr

*Attempt **any 5 questions**. Each question carries equal marks. Write your answer in detail as far as possible.*

1. Show that for any rational number q, the real number $\cos(q\pi)$ is algebraic. Hint: consider $e^{i\pi q}$
2. Let α be an algebraic complex number. Give the definition of the minimal polynomial f_α of α over $\mathbb{Q}$ and prove that $\deg f_\alpha = [\mathbb{Q}[\alpha] : \mathbb{Q}]$.
3. Show that $\mathbb{Q}[\sqrt{5} - \sqrt[3]{2}] = \mathbb{Q}[\sqrt{5} + \sqrt[3]{2}]$ and compute the dimension $[\mathbb{Q}[\sqrt{5} + \sqrt[3]{2}] : \mathbb{Q}]$ justifying your answer.
4. Show that any finite extension of fields is necessarily algebraic.
5. Give the definition of contructible number and determine which among $\sqrt[3]{2}$, $\sqrt[4]{8}$ and $\sqrt{5}+\sqrt{7}$ is constructible.
6. Describe the splitting field of the polynomial $(X^4 - 5X)(X^2 + 3)$ and write down the elements of its Galois group.
7. State in its full generality the *Fundamental Theorem of Galois Theory*
(NOTE: sometimes it is also called the *Galois Correspondance Theorem*).