Tribhuvan University

# Institute of Science and Technology

Kirtipur, Kathmandu Nepal

## Final Examination 2073 September

*Subject*: **Mathematics (Field and Galois theory)** *Full Marks*: 60
*Course No.*: Math 724 *Pass Marks*: 30
*Level*: **M. Phil.(math)**/I Semester *Time*: 2:00 hr

*Attempt* ***any 5 questions****. Each question carries equal marks. Write your answer in detail as far as possible.*

1. Show that for any rational number $r$, the real number $\sin(r\pi)$ is algebraic. Hint: consider $e^{i\pi r}$
2. Let $\beta$ be an algebraic complex number. Give the definition of the minimal polynomial $f_\beta$ of $\beta$ over $\mathbb{Q}$ and prove that $\deg f_\beta = [\mathbb{Q}[\beta] : \mathbb{Q}]$.
3. Show that $\mathbb{Q}[\sqrt[3]{7} - \sqrt{2}] = \mathbb{Q}[\sqrt[3]{7} + \sqrt{2}]$ and compute the dimension $[\mathbb{Q}[\sqrt[3]{7} + \sqrt{2}] : \mathbb{Q}]$ justifying your answer.
4. Show that any finite extension of fields is necessarily algebraic.
5. Give the definition of contructible number and determine which among $\sqrt[6]{2}$, $\sqrt[4]{27}$ and $\sqrt{5} - \sqrt{3}$ is constructible.
6. Describe the splitting field of the polynomial $(X^4 - 7X)(X^2 + 3)$ and write down the elements of its Galois group.
7. State in its full generality the *Fundamental Theorem of Galois Theory* (NOTE: sometimes it is also called the *Galois Correspondance Theorem*).