Leuca2016

Celebrating Michel Waldschmidt's 70th birthday

June 13-17, 2016, Marina di San Gregorio, Patù (Lecce), Italy

Abstracts

Bounded Height Problems and Irrationality

Francesco Amoroso (joint work with P. Corvaja and U. Zannier)

In this talk we describe a quite unexpected connection between bounded height problems and irrationality. We present a general result which allows to recover (with an apperently new proof) Thue-Siegel theorem and also effective irrationality measures for some values of algebraic functions.

Transcendence and the André-Oort conjecture

Yves André

Principally polarized abelian varieties of dimension g are parametrized by the algebraic variety A_g, those with prescribed extra "symmetries" by special subvarieties of A_g, and those with maximal symmetry (complex multiplication) by special points.

The André-Oort conjecture characterizes special subvarieties of A_g by the density of their special points (this is a hyperbolic analogue of the Manin-Mumford conjecture). It has been proven last year, after two decades of collaborative efforts putting together many different areas: arithmetic reduction theory, ergodic theory, hyperbolic geometry, model theory, analytic number theory, automorphic forms...

The conjecture also has rich connections with transcendental number theory. If time permits, I will evoke six of them.

Very special values of hypergeometric functions

Frits Beukers

In 1988 J.Wolfart proved the remarkable result that certain hypergeometric functions assume algebraic values in a dense set of algebraic arguments. Very soon, some explicit examples became available. In this lecture we discuss some further examples and other remarkable special values.

Collinear CM-points

Yuri Bilu (joint work with Florian Luca and David Masser)

We call "CM-point" the points in $\mathbb{C}^2$ whose both coordinates are j-invariants of CM elliptic curves. The now classical theorem of Yves André asserts that a non-special algebraic plane curve contains at most finitely many CM-points, the special curves being the horizontal lines $y = y_0$, the vertical lines $x = x_0$ and the standard plane models of the modular curves $Y_0(N)$. In particular, this holds true for non-special straight lines, the special straight lines being the horizontal lines, vertical lines and the diagonal $x = y$, which is nothing else but the modular curve $Y_0(1)$.

This raises the following natural questions, asked by Thomas Scanlon:

(a) do there exist non-special straight lines passing through 3 (or more) CM-point?

(b) do there exist infinitely many such lines?

Question (a) was answered positively as a by-product of a joint work with Bill Allombert and Amalia Pizarro.

The main topic of the talk is the negative answer to question (b), which is a joint work with Florian Luca and David Masser. Our main tool is Pila's multidimensional generalization of André's theorem.

Relations entre multizêtas en profondeur 3

Francis Brown

C'est Michel Waldschmidt qui m'a appris les relations de double mélange pour les multizêtas lors de son cours de DEA en 2000. Ce sont des équations algébriques élémentaires dont on ne comprend pas encore la structure. J'expliquerai comment résoudre ces équations en profondeur 2 et 3.

Derivatives and Periods of (Tensor Powers of) Carlitz Modules

Dale Brownawell

This talk is meant to give an introduction to transcendence, in particular in the function field setting, and to show how, together with a result of van der Poorten's and mine extending work of L. Denis, a recent deep result of M. Papanikolas provides another instance confirming the fundamental credo of transcendence.

On the b-ary expansions of $\log(1+\frac{1}{a})$ and e.

YANN BUGEAUD

Let $b \geq 2$ be an integer and ξ an irrational real number. In a joint work with Dong Han Kim, we prove that, if the irrationality exponent of ξ is equal to 2 or slightly greater than 2, then the b-ary expansion of ξ cannot be 'too simple', in a suitable sense. Our result applies, among other classical numbers, to badly approximable numbers, non-zero rational powers of e, and $\log(1+\frac{1}{a})$, provided that the integer a is sufficiently large. It establishes an unexpected connection between the irrationality exponent of a real number and its b-ary expansion.

On the Hilbert Property for algebraic varieties

PIETRO CORVAJA

In 1892, David Hilbert proved his famous irreducibility theorem. In modern language, it can be rephrased in the following way: given an algebraic curve C defined over a number field k and a morphism $f : C \to \mathbb{A}^1$ from C to the line $\mathbb{A}^1$ admitting no rational section, the image $f(C(k))$ cannot cover the set $k = \mathbb{A}^1(k)$. In this talk we shall investigate higher dimensional extensions. While it is well-known that the affine line can be replaced by any rational variety, we shall provide examples of non-rational algebraic surfaces X such that for every morphism $f : Y \to X$, Y being another surface, admitting no rational section, the image $f(Y(k))$ cannot cover $X(k)$. This is a joint work with Umberto Zannier.

Heights on semi-abelian varieties

SINNOU DAVID

The classical Lehmer problem conjectures a lower bound for the Weil height of a non torsion element of $\mathbf{G}_m(\overline{\mathbf{Q}})$ in terms of the degree of its field of definition. Extensions of this problem to abelian varieties and more general multiplicative toruses have been studied over the last two decades which we shall review before approaching the semi-abelian case.

Binary forms of given invariant order

Jan-Hendrik Evertse

Let A be an integral domain with quotient field K of characteristic 0. We assume henceforth that A is integrally closed, and finitely generated as a $\mathbb{Z}$-algebra. Two binary forms $F, G \in A[X, Y]$ are called $\mathrm{GL}(2, A)$-equivalent if $G(X, Y) = \varepsilon F(aX + bY, cX + dY)$ for some $\varepsilon \in A^*$ and $\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in \mathrm{GL}(2, A)$. An important invariant of a binary form F is its discriminant $D(F)$, which is given by $\prod_{1 \le i < j \le n} (\alpha_i\beta_j - \alpha_j\beta_i)^2$ if $F(X, Y) = \prod_{i=1}^{n} (\alpha_i X - \beta_i Y)$. If F has its coefficients in A then $D(F) \in A$. Moreover, if $F, G \in A[X, Y]$ are two $\mathrm{GL}(2, A)$-equivalent binary forms then $D(G) = \eta D(F)$ for some $\eta \in A^*$.

It follows from work of among others Birch and Merriman (1972) that if A is the ring of S-integers in an algebraic number field then for any given $n \ge 2$ and non-zero $\delta \in A$, there are only finitely $\mathrm{GL}(2, A)$-equivalence classes of binary forms $F \in A[X, Y]$ of degree n with $D(F) \in \delta O_S^*$. This finiteness result is no longer true for the general class of domains considered in the beginning of this abstract.

Another invariant of a binary form $F \in A[X, Y]$ is its so-called invariant A-order A_F, which is a commutative A-algebra which as an A-module is free of rank equal to the degree of F. Two $\mathrm{GL}(2, A)$-equivalent binary forms have isomorphic invariant A-orders, and the invariant A-order of a binary form determines its degree and its discriminant, up to multiplication with units from A. In case the discriminant $D(F)$ is non-zero, the invariant A-order is reduced. So a consequence of the result of Birch and Merriman is that if A is the ring of S-integers of a number field then for any given reduced, commutative ring O there are only finitely many $\mathrm{GL}(2, A)$-equivalence classes of binary forms $F \in A[X, Y]$ with $A_F \cong O$.

We show that the latter result does extend to the general class of domains considered in the beginning of this abstract. Moreover, we give a uniform upper bound for the number of $\mathrm{GL}(2, A)$-equivalence classes, depending only on the domain A and the A-module rank of O.

This is partly joint work with Kálmán Győry.

Diophantine Approximations on Definable Sets and Applications

Philipp Habegger

The celebrated Pila-Wilkie Counting Theorem gives a strong bound for the number of rational points of bounded height on a set that is definable in an o-minimal structure. We discuss an analogous result that bounds the number of rational approximations of bounded height to such a set. As an application we investigate sums of roots of unity of small modulus.

On the number of torsion points on an abelian variety defined over a number field.

Marc Hindry

We propose a survey on our knowledge about the size of the finite group of torsion points $A(K)_{torsion}$ for an abelian variety A of dimension g defined over a number field K. One may ask for explicit bounds or optimal qualitative bound either for fixed g and K (Mazur-Kammienny-Merel when $g = 1$) or for a fixed A and varying K.

Elementary integration and relative Manin-Mumford

David Masser

In 1916 Hardy wrote "... no general method has been devised by which we can always tell, after a finite series of operations, whether any given integral is really elementary, or elliptic, or belongs to a higher order of transcendents."And 100 years later nothing much has changed. Every schoolgirl knows that $\int \frac{dx}{\sqrt{x(x-\lambda)}}$ is elementary; that is, it can be expressed with logarithms and exponentials. But not $\int \frac{dx}{\sqrt{x(x-1)(x-\lambda)}}$ unless $\lambda = 0, 1$. In 1981 James Davenport asserted that if f is algebraic then $\int f(x.\lambda)dx$ is elementary for at most finitely many special complex values λ (unless it is elementary for a general value of λ). I describe progress by Umberto Zannier and myself on Davenport's Assertion. The main tools are suitable generalizations of the Manin-Mumford conjectures, in their very simplest form about roots of unity, but here about torsion on families of group varieties.

Padé approximation and the irrationality of polylogarithms

Noriko Hirata-Kohno

We consider a criterion concerning with the irrationality and the linear independence related to polylogarithms. We also give a quantitative result and some generalization.

Integral constructions of Diophantine approximations to special numbers

Yury Nesterenko

There are many classical mathematical constants whose arithmetic nature is still unknown. We discuss existing methods of proving irrationality of special numbers like logarithms of algebraic numbers. To prove irrationality usually we need to construct infinite sequences of sufficiently good rational approximations to numbers under consideration. In many cases these constructions are based on properties of hypergeometric functions. Irrationality exponent of a given real number is a quantitative characteristic of irrationality. The central problem in this area is to prove the best as possible upper bounds for the exponent of irrationality of a given number. We will discuss some recent results in this direction.

Periods and isogenies

Gäel Rémond

The talk will present a series of joint works with Gaudron in which we give several explicit versions of the period and isogeny theorems.

General Parametric Geometry of Numbers

Wolfgang Schmidt

Let K be a symmetric convex body in finite dimensional Euclidean space and $K(q)$ its transforms by the q-th powers of some linear map. Given a lattice we are interested in its successive minima with respect to $K(q)$ as functions of q. Many open questions remain

On the representation of integers by binary forms

Cameron Stewart

This is joint work with Stanley Xiao. Let F be a binary form with integer coefficients, non-zero discriminant and degree at least 3. We discuss the problem of estimating the number of integers of absolute value at most Z which are represented by F. In particular we establish an asymptotic estimate for the counting function.

Multivariate transfinite diameter

William Zudilin

How to choose a nonzero polynomial with integer coefficients that have a small sup-norm on a given interval $[a, b]$? This problem has a long history and many applications in number theory including, for example, irrationality questions of mathematical constants in qualitative and quantitative forms. In my talk I will discuss a natural generalization to the case of multivariate polynomials of small norm on $[a, b]^d$, in the arithmetic context.