Exercise 1. (1) Determine the center of D_n. (2) Determine the conjugacy classes of D_n. (It might help a little to know that, if n is even, then there are $n/2+3$ conjugacy classes, and if n is odd then there are $(n-1)/2+2$. conjugacy classes.)

Exercise 2. If $n \geq 3$, show that the center of S_n is trivial. For $n \geq 4$, show that the center of A_n is trivial.

Exercise 3. Let $n \geq 1$.
(a) Let $\sigma \in S_n$ be a product of disjoint cycles c_i of length n_i. Show that σ has order $\mathrm{lcm}(n_i)$.
(b) Exhibit an element of order 6 in S_5.
(c) Exhibit some $n \geq 1$ for which S_n contains an element of order $> n^2$.

Exercise 4. Let $n \geq 1$ and let $\sigma \in A_n$. Let C denote the conjugacy class of σ in S_n. So we have $C = \{\tau\sigma\tau^{-1} : \tau \in S_n\}$.
(a) Show that $C \subset A_n$.
(b) Show that either C is a conjugacy class of A_n or it is a union of two conjugacy classes.
(c) Show that C is a conjugacy class of A_n if and only if there is an *odd* permutation in the S_n-centralizer of σ.

Exercise 5. Let $n \geq 1$. Show that $\mathbf{Z}/n\mathbf{Z}$ admits n distinct 1-dimensional representations.

Exercise 6. Let $H = \{(1), (1\,2)(3\,4), (1\,3)(2\,4), (1\,4)(2\,3)\}$ in A_4.
(a) Write down the multiplication table of the elements in H and deduce that H is a normal subgroup of A_4 that is isomorphic to Klein's 4-group.
(b) Show that H is the commutator subgroup of A_4.
(c) Show that A_4 admits three distinct 1-dimensional representations.

Exercise 7. Show that every finite group G has a faithful representation, i.e., there is a representation (π, V) such that the homomorphism $\pi\colon G \to GL(V)$ is injective. (One can rephrase this as "every finite group is a subgroup of $GL(n, \mathbf{C})$ for some n".)

Exercise 8. Let $A : \mathbf{C}^2 \longrightarrow \mathbf{C}^2$ be the map given by $A(x, y) = (y, x)$.
(a) Show that $A^2 = \mathrm{id}$ and let G denote the group $\{I, A\}$.
(b) Write this representation $r : G \longrightarrow \mathrm{GL}_2(\mathbf{C})$ as a product of two 1-dimensional representations.