**REPRESENTATIONS OF FINITE GROUPS**

EXERCISE

Solve the following exercise from Serre's book.
Hint for question (a) : don't forget that the space of class functions has a nice orthonormal basis...

EXERCISES

**10.5.** Let $\chi$ be an irreducible character of a group G.

(a) Suppose that $\chi$ is a linear combination with positive real coefficients of monomial characters. Show that there exists an integer $m \geqslant 1$ such that $m\chi$ is monomial.

(b) Take for G the alternating group $\mathfrak{A}_5$. The corresponding permutation representation is the direct sum of the unit representation and an irreducible representation of degree 4; take for $\chi$ the character of this latter representation. If $m\chi$ were induced by a character of degree 1 of a subgroup H, the order of H would be equal to $15/m$, and $m$ could only take the values 1, 3, 5, 15. Moreover, the restriction of $\chi$ to H would have to contain a character of degree 1 of multiplicity $m$ (observe that G has no subgroup of order 15). Conclude that $\chi$ cannot be a linear combination with positive real coefficients of monomial characters.