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Non-Wieferich primes and Euclidean algorithm in number fields

Srinivas Kotyada and
Subramani Muthukrishnan

An odd prime p is said to be a non-Wieferich prime with respect to the base a if

$$a^{p-1} \not\equiv 1 \pmod{p^2}. \qquad (1)$$

The following are some important results on non-Wieferich primes.

Theorem 1 *(J.H. Silverman [1]) For any fixed* $\alpha \in \mathbb{Q}^{\times}, \alpha \neq \pm 1$, *and assuming the abc conjecture, card* $\left\{p \leq x : \alpha^{p-1} \not\equiv 1 \pmod{p^2}\right\} \gg_{\alpha} \log x \quad$ *as* $\quad x \to \infty$.

Theorem 2 *(M. Ram Murty, H. Graves [2]) For any* $a \geq 2$ *and any fixed* $k \geq 2$, *there are* $\gg \log x / \log\log x$ *primes* $p \leq x$ *such that* $a^{p-1} \not\equiv 1 \pmod{p^2}$ *and* $p \equiv 1 \pmod{k}$, *under the assumption of abc conjecture.*

Recently, the authors generalized the notion of non-Wieferich primes to algebraic number fields [3] and proved the following theorems.

Theorem 3 *[3] Let $K = \mathbb{Q}(\sqrt{m})$ be a real quadratic field of class number one and assume that the abc conjecture holds true in K. Then there are infinitely many non-Wieferich primes in O_K with respect to the unit ε satisfying $|\varepsilon| > 1$.*

Theorem 4 *[3] Let K be any algebraic number field of class number one and assume that the abc conjecture holds true in K. Let η be a unit in O_K satisfying $|\eta| > 1$ and $|\eta^{(j)}| < 1$ for all $j \neq 1$, where $\eta^{(j)}$ is the jth conjugate of η. Then there exist infinitely many non-Wieferich primes in K with respect to the base η.*

By computing non-Wieferich primes in number fields the authors proved that certain cyclic cubic fields of class number one are Euclidean (see [4] for details).

References

[1] J. H. Silverman *Wieferich's criterion and the abc-conjecture,* J. Number Theory. 30 (1988), no. 2, 226 – 237.

[2] H. Graves, M. Ram Murty, *The abc conjecture and non-Wieferich primes in arithmetic progressions,* J. Number Theory 133 (2013), 1809 –1813.

[3] K. Srinivas, M. Subramani, *Non-Wieferich primes in number fields and abc conjecture*, Czechoslovak Mathematical Journal, 68, no. 2, 2018, 445-453.

[4] K. Srinivas, M. Subramani, *A note on Euclidean cyclic cubic fields.* JRMS, 33, vol.2, 2018, 125-133.

Srinivas Kotyada
Professor, Institute of Mathemcmical Sciences
HBNI, CIT Campus, Taramani
Chennai - 600 113, India.
email: srini@imsc.res.in

Subramani Muthukrishnan
Harish Chandra Research Institute, HBNI
Chhatnag Road, Allahabad - 211 019, India.
email: msubramani@hri.res.in