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Computing isomorphism classes of abelian varieties over finite fields

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Deligne proved in [Del69] that the category of ordinary abelian varieties over a finite field $\mathbb{F}_q$ is equivalent to the category of free finitely generated $\mathbb{Z}$-modules endowed with an endomorphism satisfying certain easy-to-state axioms. In [CS15] Centeleghe and Stix extended this equivalence to all isogeny classes of abelian varieties over whose characteristic polynomial of Frobenius does not have real roots under the assumption that q is a prime number. Let C be an isogeny class in source category of Deligne or Centeleghe-Stix' equivalences and let h be the Weil polynomial associated to C. Assume that h is squarefree and denote by K the $\mathbb{Q}$-algebra $\mathbb{Q}[x]/(h)$. Put $F = x \bmod h$ and $V = q/V$ and consider the order $R = \mathbb{Z}[F, V]$ in K. Using Deligne's and Centeleghe-Stix' equivalences we obtain the following:

Theorem 1 *[Mar18b] There is an equivalence between the category of abelian varieties in C and the category of fractional R-ideals in K.*

In particular, we get a bijection between the isomorphism classes of abelian varieties in C and the *ideal class monoid* of R. There are well known algorithms to compute the group of invertible ideal classes of an order but not much can be found in the literature about non-invertible ideals. In [Mar18a] we explain how to effectively compute representatives of all ideal classes for any order in a product of number fields, allowing us to compute the isomorphism classes of the abelian varieties in C. Moreover, using results of Howe from [How95], in the ordinary case we are able to translate the notion of dual variety, polarizations and automorphisms (of the polarized abelian variety) in the category of fractional R-ideal and we provide algorithms to compute them, see [Mar18b].

References

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